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PAPER READ BEFORE THE SOCIETY 1954-55.

I.—KANT'S MATHEMATICAL ANTINOMIES.

THE PRESIDENTIAL ADDRESS.

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I am choosing as the subject of my Presidential Address an old-fashioned topic, but I think that it is one which is still of some interest. The arguments which Kant used in these antinomies themselves (as distinct from his attempted solutions of them) are practically independent of his own doctrine of transcendental idealism. They are such as tend to occur to any intelligent person who reflects on these questions, and their effects are somewhat disturbing. I am inclined to doubt whether the accepted analysis of infinity and continuity in pure mathematics, though it is a wonderful intellectual achievement in its own place, has very much to contribute to the solution of such difficulties about the extension and duration of actual things and processes. It seems to me, therefore, that no excuse is needed for considering these antinomies once again.

There are two mathematical antinomies. The first is concerned with the infinite or finite extension of the world in space and with the infinite or finite duration of the world's history backwards in time. The second is concerned with the infinite or finite divisibility of matter in respect of its spatial extension.

(1) **THE FIRST ANTINOMY.** The thesis of the first antinomy is that the world had a beginning in time and is limited in space. The antithesis is that it had no beginning in time and is unlimited in space, being infinite both in its duration backwards and in its extent. Kant does not

attempt to prove either the thesis or the antithesis directly. What he claims to do is to refute in turn the antithesis and the thesis. On the assumption that the two are collectively exhaustive alternatives, a refutation of either would be an indirect proof of the other. I will now consider in turn the argument as applied to duration and the argument as applied to extension.

(1.1) *The Argument concerning Duration.* (I) *Refutation of Antithesis.* If the world never began, it must have been going on for an infinite time up to any event in its history that we choose to take, *e.g.*, the Battle of Waterloo. This means that an infinite sequence of successive phases in its history would have elapsed before the Battle of Waterloo. Kant says that this is impossible.

The only reason which he gives is this: "The infinity of a series consists in the fact that it can never be completed through successive synthesis." (A.427, B.455). Before commenting on this it will be well to consider the additional remarks which Kant makes in his *Observations on the Thesis* (A. 430-434, B. 458-462). He says there that he might have made the argument easier for himself by taking a different definition of an infinite quantum, which is very commonly accepted, but which he regards as unsatisfactory. The unsatisfactory definition comes to this. A quantum is infinite if it contains a number N of equal units (*e.g.*, years or cubic feet) adjoined to each other, where N is such that no number greater than N is possible. On this definition the very notion of an infinite quantum could be rejected off-hand as self-contradictory. For there can be no number N such that a greater number is impossible; the sequence of integers has and can have no greatest term. In a footnote (A. 433, B. 461) Kant states what he considers to be the mathematical notion of an infinite quantum. It comes to this. A quantum is infinite if for every number N it contains a greater number of equal adjoined units than N . Thus, *e.g.*, a duration is infinite if for every number N it contains a greater number of successive years than N .

This definition does not, like the other, rule out the possibility of an infinite quantum at the start by making it involve the impossible notion of a greatest cardinal number. But it is not the definition which Kant states in the text itself. This runs as follows: "The true transcendental concept of infinitude is this, *viz.*, that the successive synthesis of units required for the enumeration of a quantum can never be completed". In the footnote he says that this entails the "mathematical concept of the infinite", which I have given in my own words above. In the text he says that it follows from the "transcendental concept" that "an infinity of actual successive states leading up to a given moment . . . cannot have elapsed".

The question is whether this "transcendental concept" of the infinity of a quantum is satisfactory, and whether it really does entail the consequences which Kant says it does.

It seems to me to be unsatisfactory, for the following reason. It drags in a reference to an operation to be performed by someone in a sequence of steps, and it defines the infinity of a quantum in terms of the impossibility of completing that operation in any time, however, long.

In order to take the simplest possible concrete case, let us apply the definition to the notion of a straight line L , which has one end A , but is supposed to stretch out to an infinite length in one direction from A . I take it that what Kant would say is this. Suppose you were to lay down a unit measure of length, *e.g.*, a foot-rule AB , with its end A coinciding with the end A of the line L , and its end B coinciding with another point of the line L . Then turn AB through 180° about its end B , so that its end A now coincides with a point 2 feet along the line L . Then turn it in the same direction through 180° about its end A , so that its end B now coincides with a point 3 feet along the line L . And so on. Then, if and only if the line L is of infinite length, there will always be a part of it which remains unmeasured however long you may continue this process.

Now the reference to the series of operations with the

measuring-rod, and to the time taken in performing it, seems to me to be irrelevant. It does not really matter that it would take a certain time to turn the foot-rule on each successive occasion. The relevant fact is that, even if each turn could be performed literally *instantaneously* and there were no *interval* between successive turns, no *number* of such adjoined units would constitute a line as long as *L*. Here all reference to temporal sequence has disappeared. The only way left to drag it in would be by the absurd expedient of saying that number involves *counting*, and that counting is essentially a sequence of acts.

The question that remains is whether this definition of an infinite quantum makes it impossible that an infinite time should have elapsed up to a given event.

I can only suppose that what Kant had at the back of his mind is something like the following. The successive phases of the history of the world, *e.g.*, the contents of successive years of world-history, up to a given moment, *e.g.*, up to January 1st, 1900, *have* automatically *synthesized themselves* by adjunction to a completed whole, *viz.*, the total history of the world up to that date. (We can ignore the fact that the history of the world continued after that date. For the present purpose it would do just as well if the universe had been completely annihilated on January 1st, 1900). So, Kant would say, the history of the universe up to that date does *not* answer to the definition of being infinite in duration. For, if it did, *no* successive adjunction of one year to another, however often repeated, would give a duration as long as the actual duration of the world up to a given date. Yet it fact that is exactly what has automatically happened through the lapse of successive years. Therefore the past duration of the world must be *finite*, and therefore the world must have had a *beginning*.

It seems to me plain that there is something wrong with this argument, though I think that there is at the back of it a genuine and important distinction between time and space. The distinction is this. In the case of an *extended* object any synthesis that may be performed is *extrinsic* to

the quantum itself. The synthesis consists, *e.g.*, in seeing different parts of the object successively, remembering at each stage what one is no longer seeing, and joining in imagination what one is now seeing to what one is now only remembering. Or, again, it may consist, *e.g.*, in successively laying down a measuring-rod and keeping a record of the number of times one has turned it through 180° or shifted it parallel to itself. It is always taken for granted that the *spatially* extended object exists at every moment as a complete whole, quite independently of the process of synthesis and of the stage which that has reached at any moment. But, in the case of a *temporal* process, successive synthesis seems to be *intrinsic* to the process. The history of the world just consists of the totality of the phases which have successively *adjoined themselves* to each other in a series; and process just *consists in* such successive synthesis of new phases to old ones.

Now this is an important peculiarity of time and duration. But just for that reason it seems to me that Kant makes here an unjustified use of his definition of infinity in terms of the impossibility of completion by successive synthesis of adjoined units. Obviously the successive synthesis of units which is contemplated in this definition is the kind of *extrinsic* synthesis which we perform when we survey a spatially extended object part by part, or when we measure it by repeatedly laying down a rod, and so on. What Kant does here is to substitute for this the *intrinsic* synthesis which is uniquely characteristic of temporal process, and then to apply the definition, which is in terms of *extrinsic* synthesis, to show that the history of the world cannot be infinitely long *a parte ante*. It seems to me that the substitution of this new and unique sense of "successive synthesis" for the old one has robbed the definition of all meaning.

I think that one can see the invalidity of the argument and the doubtfulness of the conclusion, if one imagines oneself starting, as before, from a given date in the world's history, *e.g.*, January 1st, 1900, and measuring *backwards*

from it in years. Here we can apply the original definition, for it is now *we* who are performing an *extrinsic* synthesis of units. Now here either of the following two alternatives seems to be equally conceivable. (i) That there is a number N , such that further than N years back from January 1st, 1900, neither matter nor minds nor anything else existed. (2) That for *every* number N there was matter or minds or something else existing further than N years back from January 1st, 1900.

I suspect that the conclusion of the Thesis, *viz.*, that the world must have had a beginning, may derive a certain plausibility from a kind of spatial picture or metaphor which is very hard to avoid. One tends to think of the history of the world by analogy with a strip of tooth-paste which is being continually and steadily pressed out of a tube. One then takes any actual phase in the world's history as analogous to a cross-section of this strip, and one takes the length of the strip between the mouth of the tube and this cross-section as representing the duration of the world's history up to that particular phase. Then one asks oneself: How could the strip ever have got to this, or to any other, determinate point if the mouth of the tube had been infinitely remote?

However seductive this picture may be, one can see that it is nonsensical by making the following elementary reflexion. It is sensible to ask: How *fast* is the paste coming out of the tube? And it is sensible to say that it might be coming faster or slower. But it is meaningless to ask: How quickly do the contents of successive years succeed each other? And it is meaningless to say that they might do so faster or slower. It is most important to realise that time is something absolutely unique, and that no metaphors from the movements or other changes of particular things can be anything but misleading if used to elucidate the notion of "absolute becoming", which they all presuppose.

(II) *Refutation of Thesis.* In refuting the Thesis Kant begins by asserting that to say that the world had a beginning

implies the existence of empty time before the world began. He then argues that there is nothing in any moment of empty time to determine why the world should begin at that moment rather than at any other. He then tacitly assumes Leibniz's Principle of Sufficient Reason. Since there would be no sufficient reason why the world should have begun at one rather than at another moment of absolute time, it cannot have begun at any. Therefore it never began.

I cannot see that Kant is justified in saying that the supposition that the world began involves the notion of a previous empty time, if that means an existent entity of a peculiar kind. Suppose, *e.g.*, that the relational theory of time were correct, and that time and instants are logical constructions out of direct temporal relations between events. Then to say that the world had a beginning is simply to say that there was a certain event which was followed by others but was not preceded by any other event. To say that this event would "have been preceded by empty time" would come to this. It would amount to saying that it is logically possible that there should have been events which preceded the event which was *in fact* the first event.

On this relational view of time the question: "Why did the world begin when it did, and not at some earlier or later moment?" would reduce to the question: "Why did the particular event, which in fact had no predecessors, not have predecessors?" Certainly this question could always be asked, however far back from the present we suppose the first event to have been. And it certainly would not arise if there were no event without predecessors. But I cannot help doubting whether it is a significant question, except in a rather special theistic context; and in that context the only answer is: "God knows!". So I am not prepared to accept Kant's argument as a conclusive objection to the possibility that the world had a beginning.

(1.11) *Additional Comments on the First Antinomy as regards Time.* Before passing to Kant's arguments about spatial

extension I will make two general remarks about the First Antinomy in respect to Time.

(i) Kant does not explicitly distinguish the following two questions. (a) Was there, or was there not, a *first event* in the world's history? (b) Is the *duration* of the world's history backwards from any assigned phase in it *finite or infinite*? These are certainly different questions, though they are no doubt logically connected.

One logical relation between them would seem to be this. If there was a first event in the world's history, then the duration of that history backwards from any assigned phase in it is finite. But does the converse of this hold? Can we say that, if there was *not* a first event in the world's history, then the duration of that history backwards from any assigned phase in it is *infinite*? If we are prepared to press the analogy between a continuous temporal sequence of instants or instantaneous events, on the one hand, and a continuous sequence of rational fractions in order of magnitude, on the other, the answer would seem to be in the negative.

Consider, *e.g.*, the sequence of rational fractions, in ascending order of magnitude, and take, *e.g.*, the segment of it up to and including the fraction $\frac{1}{2}$. This certainly has no first term, since there is no smallest fraction. Yet one would certainly say that the segment up to and including $\frac{1}{2}$ is of finite "length", and that its "length" is in fact one half of that of the segment up to and including the fraction $1/1$. Suppose now that we are willing to press the analogy between instants and their temporal sequence, on the one hand, and rational fractions and their sequence in order of magnitude, on the other. Then we shall have to admit the possibility that the world's history had no first phase and yet that its duration backwards from the present is finite. For my own part I regard analogies between actual existents (such as events and their temporal relations) and abstract entities (such as pure numbers and their arithmetical relations) as an extremely shaky foundation for any conclusions about the former. But I mention for

what it is worth the consequence of pursuing the analogy here.

(ii) If we admit the possibility just mentioned, we have the following three alternatives about the world's history:—
 (a) A first event and therefore a finite duration backwards from the present. (b) No first event, but a finite upper limit of duration backwards from the present. (c) No first event and an infinite duration backwards from the present.

Now it seems to me that, when these alternatives are envisaged, one sees that the fundamental question is about the possibility or impossibility of a *first event* and about the possibility or impossibility of there being *no first event*. The question of finite or infinite duration backwards from the present seems to be of interest only in so far as an answer to it carries with it one or other of the alternatives, a first event or no first event.

Now (if I may make some personal confessions) I find no difficulty in supposing that the world's history had no beginning and that its duration backwards from its present phase is infinite. Nor do I find any insuperable difficulty in supposing that the world's history had no beginning, but that its duration backwards from its present phase does not exceed a certain finite limiting value. But I must confess that I have a very great difficulty in supposing that there was a first phase in the world's history, *i.e.*, a phase immediately before which there existed neither matter, nor minds, nor anything else. I note the following two autobiographical facts here for what they may be worth. (a) I have no difficulty in supposing that there might be a *last* phase in the world's history, *i.e.*, one immediately *after* which there will exist neither matter nor minds nor anything else. (b) I have no difficulty in supposing that the material world may have an *outer spatial boundary*.

Both these facts suggest that the difficulty which I feel is connected with something peculiar to *Time*, as distinct from *Space*. The first of them suggests that it is not just a psychological difficulty due to the fact that I have had no experience of absolute beginnings. For I have equally

had no experience of absolute endings. To speak more accurately, I *have* had plenty of experience of what seemed *prima facie* to be absolute beginnings, *e.g.*, when dew was precipitated; and I *have* had plenty of experience of what seemed *prima facie* to be absolute endings, *e.g.*, when a volatile liquid totally evaporated. But in spite of this I find myself taking an entirely different attitude towards the suggestion of an absolute beginning of the world's history and the suggestion of an absolute end to it.

I suspect that my difficulty about a first event or phase in the world's history is due to the fact that, whatever I may *say* when I am trying to give Hume a run for his money, I cannot really *believe in* anything beginning to exist without being *caused* (in the old-fashioned sense of *produced* or *generated*) by something else which existed before and up to the moment when the entity in question began to exist. That this principle has no trace of self-evidence when "cause" is interpreted in terms of *law*, and not in terms of *generation*, is, of course, irrelevant. When "cause" is interpreted in terms of generation I do find it impossible to give up the principle; and with that confession of the intellectual impotence of old age I must leave this topic.

(1.2) *The Argument concerning Extension.* (I) *Refutation of Antithesis.* What Kant proves, if he proves anything, is stated as follows (A. 429, B. 457): "An infinite aggregate of actual things cannot . . . be viewed as a *given* whole, nor consequently as *simultaneously* given". He proceeds without further argument to say: "The world is therefore, as regards extension in space, not infinite. . ."

Now it is surely plain that the second proposition does not follow from the first. The most obvious meaning of the first proposition is that, if the world did consist of an infinite aggregate of co-existent things, we could at no moment perceive it as such. It may perhaps be stretched to mean that we could not know conceptually that it was such. Supposing this to be true, it would not follow that the world cannot *be* an infinitely extended whole. The utmost

that would follow is that we could never *know* it to be so, if it were so.

The argument to prove the first proposition may be stated as follows. It is obvious that any extended object which can be perceived at one glance is of *finite* extent. Hence an *infinite* whole could not be presented on any *one* occasion to sense-perception. Now any whole which cannot be perceived at one glance can be cognised perceptually only by a process of *successive synthesis*, *i.e.*, by perceiving successively different parts, which are in fact adjoined to make up the whole, and holding in memory one's perceptions of the parts which one is no longer perceiving. But it is obvious that in this way one could not in any finite time cognise perceptually any infinitely extended whole.

Kant then continues as follows: "In order therefore to conceive as a whole the world which fills all regions of space, the successive synthesis of the parts of an infinite world must be viewed as completed, *i.e.*, an infinite time must be viewed as having elapsed in the enumeration of all co-existing things". (A. 428-429, B 456-457.)

This argument seems to me to be invalid. All that Kant has proved is the trivial proposition that, if the world were infinitely extended, it would take an infinitely long time to perceive successively an exhaustive set of adjoined parts of it. He then argues that, in order to *conceive* of the world as infinitely extended, one must conceive of someone as completing this infinitely long sequence of successive perceptions. But surely that is required only in order to conceive of an infinitely extended world *being perceived* by a finite observer whose field of view at any moment is limited and who therefore has to perceive very large objects piecemeal. It does not seem to be required in order to conceive of *the world as infinitely extended*. I do not see why it is necessary to bring in a reference to perception at all. But, even if the only way of conceiving an infinitely extended world were to conceive it as something which it would take an infinite time for an observer to perceive piecemeal, Kant's conclusion would not follow. For it does not

take an infinite time to *conceive of* a process which would take an infinite time to *perform*.

(II) *Refutation of Thesis*. Kant begins the argument in the same way as he began the refutation of the thesis about time. If the world be of finite extension, he says, it must be situated somewhere within an unlimited empty space. But he continues the argument differently. He does *not* argue that there is no sufficient reason why it should be in one region of absolute space rather than another, and therefore that it must be in *every* part of space if it is in *any* part of it. Instead he argues as follows. There would have to be a certain relation (presumably the relation of "being bounded by") between the world as a whole and the empty space outside it. But this, he says, would be a relation to *nothing*, and therefore no relation. So he concludes that the world cannot be limited in its spatial extension.

This is surely a very queer argument. The only way in which I can make sense of it is to re-cast it as follows. To say that the world is limited in spatial extent implies that it is situated somewhere in Absolute Space. But there can be nothing answering to the description of Absolute Space. Therefore the world cannot be limited in spatial extent. If this is what Kant meant, he might have used a similar argument to show that the history of the world cannot be of finite duration.

Supposing this to be the argument, it seems to me to be a failure. Either we accept or we reject the possibility of an entity answering to the description of Absolute Space. If we accept it, then a relation of the world as a whole to the space outside it is *not* a relation to *nothing*. It is, indeed, not a relation to any *thing*. It is a relation to a peculiar kind of non-material extended existent. Suppose, on the other hand, that we reject the possibility of Absolute Space, and accept a relational view, such as Leibniz put forward. On that view Space and regions in Space are logical constructions out of spatial relations which hold directly between bodies. On that alternative, the statement

that the world is of finite extension means simply that there is a number N , such that the distance between any two particles in the actual world is less than N units of length, e.g., less than N miles. The statement that, if the world is of finite extent, it must be situated in empty space, would have the following meaning. It would mean that, although *in fact* there is a number N , such that no two actual particles are further than (say) N miles apart, yet there is no impossibility in supposing that there *might be* particles further apart than this or than any other number of units which could be mentioned.

(1.21) *Additional Comments on the First Antinomy as regards Space.* The following further remarks seem worth making before leaving this topic.

(i) Kant naturally assumed without question that the geometry of Absolute Space, if there were such an entity, would be Euclidean. No other alternatives had been seriously contemplated or worked out in his day.

Now a feature of Euclidean geometry, which it shares with some but not with all alternative geometries of homaloidal space, is that the straight line in it is an *open* sequence of points extending indefinitely in both directions. In some, but not all, alternative systems of geometry for homaloidal space, the straight line is a *closed* sequence of points, analogous to a great circle on a sphere. On the first alternative, Absolute Space would be intrinsically unlimited in all directions, and Kant naturally assumes this. On the second alternative, Absolute Space would be intrinsically finite. There would be a kind of natural maximum of *length*, as there is in ordinary Euclidean geometry a natural maximum of *angular deviation*, viz., the angle through which a line would have to be turned about one end in order to bring it back into coincidence with itself after completing a circle.

It is idle to speculate on what Kant would have said about such alternative possibilities. But it is important for us to notice that the spatial finitude of the material world

would be in a different logical position according to whether the geometry of nature is supposed to be of the "open" or of the "closed" type. If it is of the closed type, the material world *must* be of finite extent. The only alternatives would be: (a) that there are material particles which are at the intrinsically maximal distance apart, or (b) that every pair of material particles are at less than that distance apart. If, on the other hand, the geometry of nature is of the open type, the material world might be either of finite or of infinite extent. Relative to the "closed" alternative the finitude of the material world is necessary; relative to the "open" alternative it is contingent. But it is important to notice that the necessity of finitude on the first alternative is *only* a relative necessity. For, if it be a fact that the geometry of nature is of the closed type, it is a contingent fact and the evidence for it would be empirical. What can be said, however, is this. The evidence for the geometry of nature being of the closed or of the open kind would consist of empirical facts of a *higher order of generality* than the facts which would serve as evidence for the world being finite or being infinite in extent *given that* the geometry of nature is of the open kind.

(ii) It is important to notice that Kant never makes the *infinity* of Absolute Time or of Euclidean Absolute Space an objection to their possibility. Hence he cannot have held, as many philosophers (*e.g.*, Hegel) have done, that there is some kind of logical contradiction in the notion of infinite quanta as such. This is, indeed, pretty clear from his observations on the Thesis of the First Antinomy, which I have already quoted. He refuses to make things easy for himself by using a certain common, but mistaken, definition of infinity, *viz.*, one which involves the self-contradictory notion of a greatest cardinal number. The implication is that he regarded what he calls in the footnote (A. 433, B. 461) "the mathematical concept of the infinite", as free from contradiction.

(iii) Kant here produces no independent arguments for or against the possibility of Absolute Space or Absolute

Time, though the subject is one to which he had given much thought and about which he had reached different conclusions at different stages in his development. Here he is concerned with the extension of the *material world*, and with the duration *a parte ante* of it and of any minds which there may be in it. But, as we have seen, he does use as a premiss in one of his arguments that empty space outside the boundaries of the material world would be "nothing". Now the only ground that he gives for this is in footnote (b) to B. 457. There he simply reasserts the doctrine of the *Aesthetic* that Space and Time are merely forms of intuition.

Now there are two things to be said about this. (a) The arguments in the Antinomies are supposed to be such as would occur naturally to any intelligent man, with philosophic interests and training, who reflects and reasons on these topics. They ought not, therefore, to involve a premiss for which the only support is a special doctrine of Kant's critical philosophy. (b) Later on Kant uses the contradictory results of the mathematical antinomies to *support* his doctrine that Space and Time are merely forms of intuition. This is circular, if that doctrine is the only ground for accepting a certain premiss which is used in one of the arguments in these antinomies.

(2) THE SECOND ANTINOMY. The thesis of the Second Antinomy is that every compound substance consists of simple parts, and therefore that every substance is either itself simple or is composed of a set of parts which are simple. The antithesis is that no compound substance is composed of simple parts, and therefore that there are no simple substances. Kant is here confining his attention to material objects in space. The alleged proof that a human soul must be a simple substance is considered elsewhere, *viz.*, in the Section entitled *The Paralogisms of Pure Reason*.

(I) *Proof of Thesis*. The argument turns on a fundamental assumption, which Kant states as follows: "Com-

position, as applied to substances, is an *accidental* relation, independently of which they must still exist as self-subsistent entities". (A. 435, B. 463). I think that what this comes to is the following. If *S* be a genuine substance, then it is logically possible that it should have been *the only* genuine substance; *i.e.*, the existence of any genuine substance is logically independent of the existence of any other substance.

The other premiss in Kant's argument is that a compound substance would be a whole, composed of a set of parts each of which is itself a substance.

If we combine these two premisses, we see that it follows that what is called a "compound substance" cannot be a genuine *substance*. For the existence of any whole is logically dependent on the existence of its parts. The existence of a compound substance would therefore be *logically dependent* on the existence of the parts of which it is composed, and those parts would be themselves substances. But the existence of any genuine substance is *logically independent* of the existence of any other substance. It follows that the expression "compound substance" involves a contradiction. If there are any genuine substances, they must all be simple; what is called a "compound substance" is not really a substance, but is simply an aggregate of inter-related simple substances.

I think it must be admitted that we are entitled to draw from the premisses and the definitions the hypothetical conclusion: "If there are any genuine substances, they are none of them compound substances". But we are not entitled to draw the categorical conclusion: "There *are* genuine substances, and they are all simple." For it might be that there is nothing in the world answering to the conditions which Kant lays down for a genuine substance.

There are two points worth noting in Kant's *Observations on the Thesis* (A. 438, B. 466). (i) He says that this argument would not apply to Absolute Space or Absolute Time. The reason is that they would not be wholes composed of accidentally associated parts. In Absolute Space, *e.g.*, the

whole would be logically prior to the sub-regions which are its parts; it would not be related to them as a stack of bricks is to the individual bricks which together compose it. Nor would the various sub-regions be related to each other as the various bricks which together compose a stack are inter-related. For each brick might have existed even if none of the others had done so. But each region of Space presupposes Space as a whole, and therefore all the other sub-regions of Space.

I think that this contrast is correct and important. But it raises the question whether the material world (or rather the old-fashioned "Luminiferous Ether", in which material particles are perhaps vortices) may not really be a whole of the same kind as Absolute Space would be, and not an aggregate of accidentally inter-related simple substances.

(ii) At the end of these *Observations* Kant remarks that the Thesis is really the principle with which Leibniz begins his *Monadology*. That remark is true, so far as it goes. But it seems to me that there is one important difference. Leibniz tried to make us see that the notion of a whole composed of parts, which are themselves composed of parts, which are themselves composed of parts . . . , and so on without end, involves a regress which is vicious. Kant shows merely that a "compound substance" would not answer to his definition of a "substance".

I must confess that I cannot but find myself agreeing with Leibniz here. But I am well aware that many contemporary philosophers, at least as intelligent and acute as myself, profess to find no difficulty in the regress which Leibniz felt to be obviously vicious. That there is no formal contradiction in it, is, I think, plain. But that does not seem to me to settle the question.

(II) *Proof of Antithesis.* The Antithesis may be divided into two assertions, which we will call (1) and (2). Assertion (1) is that no composite thing is made up of simple parts. Assertion (2) is that there can be no simple existents in the world. We will take them in turn.

Assertion (1). Kant begins by saying that the notion of a compound substance applies only to extended objects. For a compound substance is an aggregate of *externally* related parts, and it is only in space that external relations are possible. We must therefore think of each compound substance as occupying a volume in space.

The next step is this. If a whole occupies a volume, every part of it must occupy some part of that volume. But the parts of a volume are themselves volumes. Therefore, if a compound substance consisted of simple parts, each of these simple parts would occupy a volume.

So far there is no difficulty. But, in order to be fair to Kant, I will quote his own words for the next step. “. . . Everything real which occupies a volume contains in itself a manifold of constituents external to each other, and is therefore composite. And . . . a real composite . . . is made up . . . of substances. . . .” (A. 436, B. 464). So what Kant asserts is that each of the allegedly *simple* substances, of which a body is alleged to be composed, would have to be a *compound* substance, in order to occupy the volume which it must occupy if it is to be part of that body. Now that is of course a contradiction. So he concludes that an extended substance cannot consist of simple parts. But all compound substances are extended. Therefore no compound substance can consist of simple parts.

Before commenting on the main argument I will remark that I do not accept Kant's statement that the notion of a compound substance applies only to extended objects. I can quite well conceive of an unembodied mind, and such a mind might fairly be called an unextended substance. I can also quite well conceive of a group of such minds, communicating with and influencing each other telepathically, and closely inter-related by certain emotional relations, common interests, etc., Such a group of closely inter-related unembodied minds might fairly be called a compound spiritual substance, though not of course a compound *mind*. And it would be unextended.

Passing now to the case of extended compound sub-

stances and to Kant's argument about them, I think that there is a fallacy. The phrase "to occupy a volume" is ambiguous. It needs to be re-defined according to the various alternative views which may be held about the nature of bodies. If we are to be fair to the Thesis, we must remember that, on its view, a body will just be a collection of intimately aggregated *unextended* particles, *e.g.*, mass-points or centres of repulsive or attractive force. Next, we must remember that to "occupy a volume", in the case of an unextended particle, can only mean to *fall within* that volume. It cannot possibly mean to *fill* that volume. Kant does not say whether the Thesis supposes a finite body to consist of a finite, or of an infinite, number of unextended particles. So I will consider each of these alternatives in turn.

(i) *The finite alternative.* If a body B consists of a *finite* number of unextended particles, it can occupy a region V only *discontinuously*. What this would come to is the following. Every particle of B falls within some sub-region of V and every sub-region of V which exceeds a certain small volume v contains at least one particle of B . Some sub-regions of V , which are smaller than v , would fail to contain any particle of B . I do not see the slightest objection to the view that every body occupies the region which it does occupy *only discontinuously*.

(ii) *The infinite alternative.* What I have to say under this head presupposes that we can safely apply the notions of infinity and continuity, developed in pure mathematics for real numbers and sequences of such numbers, to concrete particular existents in the space and time of nature. As I have already remarked, I do not feel at all sure that this is legitimate. Let us, however, suppose for the sake of argument, that it is.

Then we could combine the view that a body consists of *unextended* particles with the view that it occupies a volume *continuously*, by assigning a high enough order of infinity to the number of particles of which the body consists. I should say that a body B *continuously* occupies the

region V , if the following two conditions are fulfilled. (a) Every sub-region of V , however small, contains at least one particle of B . (b) Every particle of B falls within some sub-region of V . There is no doubt that these conditions could be fulfilled, provided that the number of particles composing the body is as great as the number of points in the number-continuum.

The place at which Kant's argument goes wrong is where he says that "everything which occupies a volume contains in itself a manifold of constituents external to each other, and is therefore composite". He here uses "occupy" in the sense of *fill*, and not merely in the sense of *fall within*. An unextended particle could not occupy a volume in the former sense; it could do so only in the latter. It seems to me that he failed to recognise that the supposition that a body is composed of *unextended* particles is compatible with at least the two following alternatives, neither of which is intrinsically impossible. (a) That the number of unextended particles composing a finite body is *finite*, and that the body occupies a volume *discontinuously*, as a crowd occupies Trafalgar Square. (b) That the number of unextended particles composing the finite body is *infinite*, and that an aggregate of a sufficiently great infinite number of unextended particles may *continuously* occupy a volume, although no individual particle can do more than fall within a volume.

Assertion (2). Finally, we come to the second assertion of the Antithesis. This is much stronger than the first. It says that there are no simple substances at all, and not merely that no extended substance is composed of simple substances.

We need not spend much time on this. For in his proof (A. 437, B. 465) Kant explains that he is not really claiming to establish this. He claims only to show that "the existence of the absolutely simple cannot be established by any experience or perception, outer or inner". This is an entirely different assertion, and I have no wish to question it.

CONCLUDING REMARKS. A complete treatment of our subject would require a discussion of Kant's solution of the two Mathematical Antinomies. This well deserves serious consideration for its own sake, but I cannot undertake it here. Instead I will conclude with some general remarks on Antinomies and their solution, which will, I hope, show that the omission is justifiable.

The general notion of an antinomy may be described as follows. You have two sentences of the form " S is p_1 " and " S is p_2 ". These seem to express propositions which are mutually exclusive and collectively exhaustive, *i.e.*, such that both of them cannot be true and both of them cannot be false. Yet it seems that both of them can be *proved* by cogent reasoning from indubitable premisses, or that both of them can be *disproved* by such reasoning from such premisses.

Now, assuming that the reasoning on both sides of an antinomy really is cogent, and that the premisses really are *prima facie* indubitable, a "solution" is plainly called for, and there are two and only two possible types of solution.

(i) Suppose that you seem to have *disproved* both that S is p_1 and that S is p_2 , although they seem to be *collectively exhaustive* alternatives. Then the solution may be that S does not really have the determinable characteristic P , under which p_1 and p_2 are the two collectively exhaustive determinates. You have tacitly assumed that it has that determinable characteristic. Once you give up that assumption you will have no difficulty in admitting that it is false *both* that S is p_1 and that S is p_2 . If, *e.g.*, the world is not really temporal at all, then we can admit that it is equally false to say that there was a *first* event in its history and to say that *every* event in its history had a predecessor.

(ii) Suppose that you seem to have *proved* both that S is p_1 and that S is p_2 , although they seem to be *mutually exclusive* alternatives. Then the solution may be that the name " S " is ambiguous, and that it stands for a different subject in the two sentences. In that case the two propositions are really of the forms S_1 is p_1 and S_2 is p_2 , and there may be

no difficulty in admitting that both may be true. Suppose, *e.g.*, that the word "man" may mean either (*a*) man as he appears to himself on introspection, or (*b*) man as he is in himself. Then the two sentences: "A man is free in some of his actions" and "A man is completely determined in all his actions" are ambiguous. If you substitute one sense of "man" in one of them and the other sense of "man" in the other, each may express a true proposition.

Kant adopts the first type of solution for the mathematical antinomies and the second for the dynamical. But, if what I have contended in this paper is correct, there is no need to adopt either of these expedients in connexion with the topics which we have been discussing. For it appears that the arguments are not cogent or the premisses are not even *prima facie* indubitable in either the thesis or the antithesis of either of the mathematical antinomies. If so, there is no genuine antinomy and therefore no call for a solution.
